Designing ultrafast mirrors by minimizing the energy of phase distortions in the frequency domain

Jonathan R. Birge and Franz X. Kaertner
Department of Electrical Engineering and Computer Science,
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
birge@mit.edu

Abstract: The direct optimization of complex filter error—modulo zeroth- and first-order phase—is proposed as an alternative to GDD optimization. In the appropriate norm, this is equivalent to minimizing the error energy for a given input.

OCIS codes: (320.5520) Ultrafast optics, pulse compression; (310.6860) Thin films, optical properties

1. Introduction

Optical thin-film mirrors are critical components of many modern ultrafast optical systems and communications devices. In some situations a specific group delay dispersion (GDD) profile is desired, such as in dispersion compensating mirrors [1], and in others the dispersion is made as small as possible to minimize its effect on short pulses. In either event, phase sensitive filters have historically been designed by numerically minimizing a weighted integral of reflectance and GDD error.

The concept of GDD arises in the context of the slowly varying envelope approximation, where the relative bandwidth is, by definition, assumed to be small enough such that dispersive broadening is well described by the local second-order phase. It is not always clear, however, how to ascribe physical meaning to the integral of squared phase derivatives over a wide bandwidth where a series approximation is not valid. Optimizing integrated GDD error would minimize the mean distortion of a set of *independent* sources each of narrow bandwidth (*e.g.* WDM channels). However, in the case where a wide bandwidth of frequencies will be coherently interfered (as with a laser pulse), mean squared GDD error may not be an ideal measure of performance.

Consider, for example, a dispersion compensating mirror used to compress an ultrashort pulse. The dominant nonideality in such mirrors is the creation of satellite pulses caused by an impedance mismatch at the mirror surface. It can be shown [2] that a spurious reflection of magnitude a at a relative delay τ causes a GDD error proportional to $a\tau^2$ (to lowest order in a). In many applications, however, it is simply the energy contained in any satellite pulses that is of relevance, not their delay. However, the quadratic scaling with delay means that GDD error is not a monotonic function of satellite pulse energy, and thus GDD minimization will converge to a suboptimal solution in this sense.

One avenue around these issues is to optimize the mirror in the time domain [3]. However, thin film filters are most naturally analyzed in the frequency domain (where they are diagonalized) and thus time domain optimization presents its own difficulties. Requiring one FFT per wavelength at each optimization step (to compute the gradient) involves significant computation, and operating in the time domain renders analytic gradients of merit functions infeasible. Thus, a single optimization step will require an effort that scales as $O(n_\lambda \log n_\lambda)$ with wavelength count n_λ and $O(n^2)$ with layer count n. This is significantly greater than the $O(n_\lambda)$, $O(n)$ complexity using analytic gradients in the frequency domain, notwithstanding the potentially slower convergence rate from using finite difference gradients.

we propose a method to optimize a reflective filter in the frequency domain in terms of a simple quantity we refer to as *phase distortion energy* (PDE). This approach expresses both reflectivity and phase errors in units of energy, putting them on an equal footing, thus facilitating weighting between them. We believe this approach will yield improved results for many applications in communications and ultrafast optics. Furthermore, the method avoids the costly evaluation of GDD and GDD gradients during optimization [4].

2. Derivation of Phase Distortion Energy

Let the ideal GDD curve of a given design be given by $\hat{D}_2(\omega)$, resulting in a desired complex reflection coefficient of $\hat{R}(\omega)e^{i\left(\hat{\phi}(\omega)+\phi_0+\phi_1\omega\right)} = \hat{R}(\omega)\exp[i\iint d\omega^2\,\hat{D}_2(\omega)]$, where constants of integration have been included. The complex transfer function of our designed filter will be written similarly, but without the hats. Let the expected power spectral density of our input be given by $P(\omega)$.

Our starting point is the energy contained in the By Parseval's theorem, the total error energy is the power weighted integral of the modulus squared difference between the ideal and approximate transfer functions. In a numerical context, this will be approximated by the discrete weighted norm:

$$\begin{aligned}\text{Error Energy} &\approx \sum_i P(\omega_i)\left|R^n(\omega_i)e^{in\phi(\omega_i)} - \hat{R}^n(\omega_i)e^{in[\hat{\phi}(\omega_i)+\phi_0+\phi_1\omega]}\right|^2 \\ &= \sum_i P(\omega_i)\left[R^{2n}(\omega_i) - \hat{R}^{2n}(\omega_i)\right] + 2\sum_i P(\omega_i)[R(\omega_i)\hat{R}(\omega_i)]^n \cos[n(\phi_{\text{err}}(\omega_i)+\phi_0+\phi_1\omega_i)],\end{aligned} \tag{1}$$

For convenience, we've written the dispersion error in terms of the corresponding phase error, which will include arbitrary zeroth- and first-order phase terms as constants of integration. Thus, we've also introduced zeroth- and first-order phases that will be allowed to "float" so as to minimize (1). The power spectral density "weighting" function $P(\omega)$ should be scaled such that the summation over the set of optimization frequencies ω_i approximates a power integral.

The cosine term is problematic. First, its periodicity would impede convergence in a local gradient optimization. Second, it makes it impossible to solve directly for the optimal values of the floating phase coefficients. Because of the floating terms, however, the argument to the cosine at the end of an optimization will generally be much smaller than π (assuming a reasonably good solution is found). As such, we can replace the cosine with a series expansion. To second order in the phase error, we get an error energy of

$$\sum_i P(\omega_i)[R^{2n}(\omega_i) - \hat{R}^{2n}(\omega_i)] + \sum_i P(\omega_i)[R(\omega_i)\hat{R}(\omega_i)]^{2n}\sqrt{n}\left(\phi_{\text{err}}(\omega_i)+\phi_0+\phi_1\omega_i\right)^2. \tag{2}$$

To solve for the floating phases, we take the gradient of the second sum in (2) with respect to the coefficients ϕ_0 and ϕ_1. Setting both gradient elements to zero yields a set of coupled equations for the optimal constant and linear phases,

$$\begin{aligned}\langle 1\rangle\phi_0 + \langle\omega\rangle\phi_1 &= -\langle\phi_{\text{err}}\rangle \\ \langle\omega\rangle\phi_0 + \langle\omega^2\rangle\phi_1 &= -\langle\omega\phi_{\text{err}}\rangle,\end{aligned} \tag{3}$$

with the weighted mean operator $\langle\cdot\rangle$ defined by $\langle f\rangle \equiv \sum P(\omega_i)R(\omega_i)\hat{R}(\omega_i)f(\omega_i)$. Solving for the optimal phases and substituting back in to (2) gives us a standard merit function that is entirely a function of layer thicknesses.

$$\begin{aligned}\text{CEE} = &\sum_i P(\omega_i)\left[R(\omega_i) - \hat{R}(\omega_i)\right]^2 + \\ &\sum_i P(\omega_i)R(\omega_i)\hat{R}(\omega_i)\left[\phi_{\text{err}}(\omega_i) + \frac{\langle\omega^2\rangle\langle\phi_{\text{err}}\rangle + \omega_i\langle 1\rangle\langle\omega\phi_{\text{err}}\rangle - \langle\omega\rangle\left(\omega_i\langle\phi_{\text{err}}\rangle + \langle\omega\phi_{\text{err}}\rangle\right)}{\langle\omega\rangle^2 - \langle 1\rangle\langle\omega^2\rangle}\right]^2.\end{aligned} \tag{4}$$

The first term in (4) is the error energy due to reflectivity, and the second term is that due to phase errors.

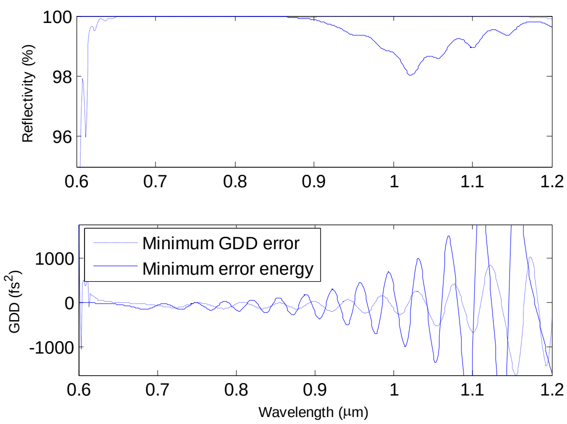

Fig. 1. Comparison of minimum energy and GDD optimized filters.

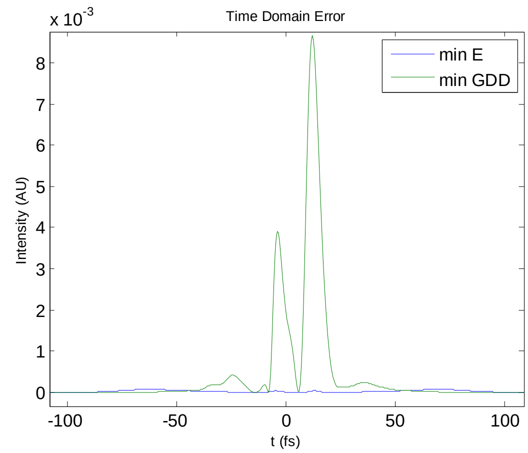

Fig. 2. Time domain error intensity of both filters in fig. 1.

To avoid phase unwrapping issues, the phase error is computed using a numerical integration of group delay. As numerical quadrature and the gradient are both linear operators, analytic gradients of the above merit functions can be computed in a straightforward manner from gradients of group delay and reflectivity [4]. While the final analytic expression is cumbersome to detail in closed form, it is not difficult to program and does not pose a significant computational burden in practice as numerical integration is linear in complexity.

3. Results

As a test case, we designed a dispersion compensating mirror intended to recompress an 8 fs pulse after traveling through 2 mm of fused silica. We first optimized on the standard basis of sum of squared GDD error, using our best guess for a relative weighting between reflection and GDD. Next, we repeated the optimization using the complex error energy merit function, with equal weighting between reflection and phase errors. The resulting mirrors are shown in Figure 1. In both cases, we weighted with the power spectral density of an 8 fs Gaussian pulse.

The GDD optimization produced a GDD curve with 6.5 times less GDD squared error as the energy minimized design. However, when looking at the effect on a pulse, the minimum energy design performs arguably better. Figure 2 shows the time domain error signal for the two filters relative to a theoretically perfect mirror. The minimum CEE design produces a significantly smaller total error signal in the time domain, a factor of 19.5. The minimum energy design has satellite pulses about 60 fs away, while the minimum GDD error design has satellite pulses contained within roughly 30 fs, but with almost 20 times more energy. This is clearly the result of the quadratic GDD scaling with delay discussed earlier. Finally, we note that the minimum energy design required no human intervention to pick weighting values.

4. Summary

We have argued that integrated GDD error is not always the most physically relevant criterion to consider when optimizing a dispersive mirror. We have developed an alternative merit function which seeks to minimize the energy of the perturbation to an input of known spectral density. This approach proved successful in the optimization of an ultrafast chirped mirror, producing a filter that introduced over an order of magnitude less distortion than with standard GDD optimization, despite having almost seven times worse GDD ripple. Further work will address the question of under what circumstances is minimizing error energy appropriate.

5. References

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