Fourier Optics Formulas

Two-Dimensional Fourier Analysis

Fourier Transform

$$\mathcal{F}\left\{g(x,y)\right\} = \iint\limits_{-\infty}^{+\infty} g(x,y)\, e^{-2\pi(f_X x + f_Y y)}\, dx\, dy$$

$$\mathcal{F}^{-1}\left\{G(f_X,f_Y)\right\} = \iint\limits_{-\infty}^{+\infty} G(f_X,f_Y)\, e^{2\pi(f_X x + f_Y y)}\, df_X\, df_Y$$

$$\mathcal{F}\left\{g(ax,by)\right\} = \frac{1}{|ab|}\, G\left(\frac{f_X}{a}, \frac{f_Y}{b}\right)$$

$$\mathcal{F}\left\{g(x-a,x-b)\right\} = G(f_X,f_Y)e^{-j2\pi(f_X a + f_Y b)}$$

$$\mathcal{F}\left\{g \star g\right\} = |G(f_X,g_y)|^2$$

Fourier-Bessel Transform

$$\mathcal{B}\left\{g_R(r)\right\} = 2\pi \int\limits_0^{\infty} r\, g_R(r)\, J_0(2\pi r\rho)\, dr$$

$$\mathcal{B}^{-1}\left\{G_0(\rho)\right\} = 2\pi \int\limits_0^{\infty} \rho\, G_0(\rho)\, J_0(2\pi r\rho)\, d\rho$$

$$\mathcal{B}\left\{g_R(ar)\right\} = \frac{1}{a^2}\, G\left(\frac{\rho}{a}\right)$$

Scalar Diffraction Theory

$$u(\mathbf{r},t) = \mathrm{Re}\left\{U(\mathbf{r})e^{-j2\pi\nu t}\right\}$$

$$(\nabla^2 + k^2)U = 0$$

Huygens-Fresnel Principle (First Rayleigh-Sommerfeld Solution)

$$U(\mathbf{r}_0) = \frac{1}{j\lambda} \iint\limits_{\Sigma} U(\mathbf{r}_1)\, \frac{\exp(jkr_{01})}{r_{01}}\, \cos\theta\, ds$$

$$U(x,y) = \frac{z}{j\lambda} \iint\limits_{-\infty}^{+\infty} U(\xi,\eta)\, \frac{\exp(jkr_{01})}{r_{01}^2}\, d\xi\, d\eta$$

Angular Spectrum

$$A\left(\frac{\alpha}{\lambda}, \frac{\beta}{\lambda}; 0\right) = \mathcal{F}\left\{U(x,y,0)\right\}\Big|_{f_X=\frac{\alpha}{\lambda}, f_Y=\frac{\beta}{\lambda}}$$

$$H_z(f_X,f_Y) = \begin{cases} \exp\left[j2\pi\frac{z}{\lambda}\sqrt{1-(\lambda f_X)^2-(\lambda f_Y)^2}\right] & \rho < \frac{1}{\lambda} \\ 0 & \text{otherwise} \end{cases}$$

Fresnel (Paraxial) Approximation

$$U(x,y) = \frac{e^{jkz}}{j\lambda z} \iint\limits_{-\infty}^{+\infty} U(\xi,\eta)\exp\left\{j\frac{k}{2z}\left[(x-\xi)^2+(y-\eta)^2\right]\right\} d\xi\, d\eta$$

$$U(x,y) = \frac{e^{jkz}}{j\lambda z} e^{j\frac{k}{2z}(x^2+y^2)} \iint\limits_{-\infty}^{+\infty} \left[U(\xi,\eta)e^{j\frac{k}{2z}(\xi^2+\eta^2)}\right] e^{-j\frac{2\pi}{\lambda z}(x\xi+y\eta)}\, d\xi\, d\eta$$

$$U(x,y) = \frac{e^{jkz}}{j\lambda z} e^{j\frac{k}{2z}(x^2+y^2)}\, \mathcal{F}\left\{U(\xi,\eta)e^{j\frac{k}{2z}(\xi^2+\eta^2)}\right\}\Big|_{f_X=\frac{x}{\lambda z}, f_Y=\frac{y}{\lambda z}}$$

$$H_z(f_X,f_Y) = e^{jkz}\exp\left[-j\pi\lambda z\left(f_X^2+f_Y^2\right)\right], \quad |\lambda f| << 1$$

Fraunhofer Approximation

$$U(x,y) = \frac{e^{jkz}}{j\lambda z} e^{j\frac{k}{2z}(x^2+y^2)} \iint\limits_{-\infty}^{+\infty} U(\xi,\eta)e^{-j\frac{2\pi}{\lambda z}(x\xi+y\eta)}\, d\xi\, d\eta$$

$$U(x,y) = \frac{e^{jkz}}{j\lambda z} e^{j\frac{k}{2z}(x^2+y^2)}\, \mathcal{F}\left\{U(\xi,\eta)\right\}\Big|_{f_X=\frac{x}{\lambda z}, f_Y=\frac{y}{\lambda z}}$$

Coherent Optical Systems

$$t_l(x,y) = \exp\left[-j\frac{k}{2f}(x^2+y^2)\right]$$

Frequency Analysis

Coherent (Linear in Amplitude)

$$H(f_X,f_Y) = P(\lambda z_i f_X, \lambda z_i f_Y)$$

$$I_i(u,v) = |h * U_g|^2 \subset \left[H \cdot \mathcal{F}\left\{U_g\right\}\right] \star \left[H \cdot \mathcal{F}\left\{U_g\right\}\right]$$

Incoherent (Linear in Intensity)

$$\mathcal{H}(f_X, f_Y) = \frac{H \star H}{H \star H\Big|_{f_X=0, f_Y=0}}$$

$$I_i(u,v) = |h|^2 * |U_g|^2 \subset [H \star H][U_g \star U_g]$$

$$\delta = 0.61 \frac{\lambda}{\mathrm{NA}}$$

Jones' Calculus

$$\mathbf{L}_{\mathrm{rotate}}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\mathbf{L}_{\mathrm{retard}}(\Delta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\Delta} \end{bmatrix}$$

$$\mathbf{L}_{\mathrm{polarizer}}(\alpha) = \begin{bmatrix} \cos^2\alpha & \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & \sin^2\alpha \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$\mathbf{L}_{\mathrm{reflect}} = \mathbf{R}\mathbf{L}^t\mathbf{L}$$